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MASSIVE TRIADIC CHERN-SIMONS SPIN-3 THEORY*

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ABSTRACT

We introduce the massive gauge invariant, second order pure spin-3 theory in three dimensions. It consists of the addition of the second order gauge invariant massless pure spin-3 action with the first order topological(generalized) Chern-Simons spin-3 term corrected with lower spin auxiliary actions which avoid lower spin ghosts propagation. This second order intermediate action completes the catalogue of massive spin-3 actions having topological structure. We also consider its spontaneous break down through the addition of the inertial spin-3 nontopological Fierz-Pauli mass term. It is shown that this non gauge invariant pure spin-3 system is the uniform generalization of linearized massive vector Chern-Simons gravity and propagates just two spin $3^{\pm}$ excitations having different masses.

1. Introduction

Recently an alternative curved topological gravitational theory in three dimension was found and discussed in detail[1]. Although massive vector Chern-Simons gravity(VCSG) propagates one massive spin-2 excitation, like topological massive

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gravity(TMG)[2] substantial differences between them occur. TMG is described by a third order local Lorentz and diffeomorphism invariant action whereas VCSG is a second order diffeomorphism invariant theory where locaSNS-HEP-l Lorentz invariance has been lost. In addition, TMG can not be broken down neither by the presence of a triadic Chern-Simons term nor by a Fierz-Pauli(FP) mass term while VCSG admits a symmetry breaking when the FP mass term is present, giving rise to a double spin-2 system having two different masses[3](similarly to what happens to the Maxwell-CS system when the Proca mass term is considered[4]). It is worth recalling that the dreibein e_{ma} is the most natural object to describe spin-2, its gauge(linearized) transformation law is given by $\delta_\lambda h_{pa} = \partial_p \lambda_a$. Self-dual massive gravity[5] [6] completes the catalogue of spin-2 massive excitations in three dimensions. It is a first order theory on flat Minkowski space having no gauge invariance.

For spin-3 it is known both the topological third order theory[7] and the self-dual first order action[8]. The former is gauge and local Lorentz invariant while the latter does not have any local symmetry left. According to what we have learnt from the spin-2 case, we will show that a gauge invariant second-order intermediate action propagating a pure spin-3^+(or spin-3^-) excitation indeed exists. It is the uniform generalization of VCSG having the novelty of making unavoidable the presence of auxiliary lower spin fields.

2. Triadic Chern-Simons spin-3 action

The action has the form

$$\begin{aligned} I &= \frac{1}{2} < h_{m\bar{a}\bar{b}} G_{m\bar{a}\bar{b}} > - \frac{\mu}{2} < h_{m\bar{a}\bar{b}} \epsilon^{mnp} \partial_n h_{p\bar{a}\bar{b}} > \\ &+ \mu < v_q \epsilon^{mnp} \partial_p h_{m\bar{n}\bar{q}} > + \frac{\alpha}{2} \mu < v_p \epsilon^{pmn} \partial_m v_n > + \frac{\beta}{2} \mu^2 < v_p^2 > \\ &+ \mu < \phi(\partial . v) > + \frac{\gamma}{2} < \phi \Box \phi > + \frac{\delta}{2} \mu^2 < \phi^2 >, \end{aligned} \tag{1}$$

wherein $h_{m\bar{a}\bar{b}}$ is the basic spin-3 carrier written in the triadic representation[9 10], symmetric and traceless in the barred Lorentz-like indices. The core of the action consists of the usual massless spin-3 second order action($\sim hG$) enlarged with a typical triadic Chern-Simons first order action for spin-3($\sim h\epsilon\partial h$) which will provide mass μ. It is invariant under the natural abelian gauge transformations $\delta_\lambda h_{p\bar{a}\bar{b}} = \partial_p \lambda_{\bar{a}\bar{b}}$(similarly to massive vectorial Chern-Simons linearized gravity). The remaining terms contain lower spin contributions stemming in the auxiliary fields v_p, ϕ and a gauge invariant coupling term linking the vector with the basic spin-3 field. As is well known[11], in $d \geq 4$, massive higher spin fields require the presence of them to make sure the non-propagation of undesirable lower spin ghosts. The basic triadic spin-3 field $h_{m\bar{a}\bar{b}}$ can be decomp

$$h_{m\bar{a}\bar{b}} = h_{\bar{m}\bar{a}\bar{b}} + \epsilon_{pac} h_{\bar{c}\bar{b}} + \epsilon_{pbc} h_{\bar{c}\bar{a}} + \frac{3}{10}[\eta_{pa} h_b + \eta_{pb} h_a - \frac{2}{3}\eta_{ab} h_p]. \tag{2}$$

The 15 independent components of $h_{m\bar{a}\bar{b}}$, are represented by the 7 components of $h_{\bar{m}\bar{a}\bar{b}}$ plus the 5 needed to describe $h_{\bar{b}\bar{c}}$ plus the last 3 which determine $h_p = h_{m\bar{m}\bar{p}}$, the unique non-vanishing trace of $h_{m\bar{a}\bar{b}}$.

Independent variations of $h_{m\bar{a}\bar{b}}$, v_p, ϕ lead to the field equations

$$E_{m\bar{a}\bar{b}} \equiv G_{m\bar{a}\bar{b}} - \mu\epsilon^{mnp}\partial_n h_{p\bar{a}\bar{b}} + \frac{\mu}{2}[\epsilon^{pna}\partial_n v_b + \epsilon^{pnb}\partial_n v_a - \frac{2}{3}\eta_{ab}\epsilon^{pmn}\partial_m v_n] = 0, \quad (3)$$

$$F^p = \alpha\mu\epsilon^{pmn}\partial_m v_n + \beta\mu^2 v_p + \mu\epsilon^{qmn}\partial_q h_{m\bar{n}\bar{p}} - \mu\partial_p\phi = 0, \quad (4)$$

and

$$G \equiv \mu(\partial.v) + \gamma\Box\phi + \delta\mu^2\phi = 0. \quad (5)$$

In order to see their dynamical content we perform a covariant analysis using the harmonic gauge defined in this case by

$$\partial_m(h_{m\bar{a}\bar{b}} + h_{a\bar{m}\bar{b}} + h_{b\bar{a}\bar{m}}) - (\partial_a h_b + \partial_b h_a) = 0. \quad (6)$$

Taking into account Eq. 2, this gauge fixing condition implies that

$$\partial_m h_{\bar{m}\bar{a}\bar{b}} = \frac{1}{5}[\partial_a h_b + \partial_b h_a - \frac{2}{3}\eta_{ab}\partial_m h_m], \quad (7)$$

and the Eq 3 becomes

$$\begin{aligned} \Box[h_{m\bar{a}\bar{b}} + h_{a\bar{m}\bar{b}} + h_{b\bar{a}\bar{m}} - \eta_{ma}h_b - \eta_{mb}h_a] \quad &- \quad \mu\epsilon^{mnp}\partial_n h_{p\bar{a}\bar{b}} \\ +\frac{\mu}{2}[\epsilon^{pna}\partial_n v_b + \epsilon^{pnb}\partial_n v_a - \frac{2}{3}\eta_{ab}\epsilon^{pmn}\partial_m v_n] \quad &= \quad 0. \end{aligned} \quad (8)$$

It is easy to check than none of the spin-2 variables have any dynamical behaviour. They do not propagate. Now, let us go to the spin-1 sector. The variables are $\rho_m h^T_{mn}, h^T_p$ and $v^T_p(\rho_p \equiv \frac{\partial_m}{\rho}, \rho \equiv (\Box)^{\frac{1}{2}})$. The spin-1 dynamical behaviour is determined by the equations $E_p \equiv E_{m\bar{m}\bar{p}} = 0, \epsilon^{mac}\partial_c E_{m\bar{a}\bar{b}} = 0$ and $F^p = 0$. In order not to have any spin-1 excitation alive we must choose

$$\alpha = -\frac{27}{6}, \quad \beta = -\frac{16}{3}, \quad (9)$$

in order to make the spin-1 inverse propagator a non-vanishinhg number.

Unfortunately this is not the last step in order to get a pure spin-3 propagation. There are still three scalar ghosts($\partial_{pn}h_{\bar{p}\bar{n}}, \partial_p h_p, \partial_p v_p$) that might propagate. This is the reason why we have to introduce the auxiliary scalar field ϕ. The key equations for the scalar sector are $\partial_p E_p = 0, \epsilon^{mac}\partial_{cb}E_{m\bar{a}\bar{b}} = 0, \partial_p F^p = 0$ and $G = 0$. One finds that

$$\gamma = 0, \quad \delta = -\frac{1}{6}, \quad (10)$$

entails the non-propagation of the whole scalar sector. Consequently, the dynamics is contained in the symmetric, traceless, transverse part of the triadic spin-3 field: $h^T_{\bar{m}\bar{n}\bar{p}}$. Its two independent components can be split into parity sensitive parts according to

$$h^T_{\bar{m}\bar{n}\bar{p}} = h^{T+}_{\bar{m}\bar{n}\bar{p}} + h^{T-}_{\bar{m}\bar{n}\bar{p}}, \tag{11}$$

with

$$h^{T\pm}_{\bar{m}\bar{n}\bar{p}} \equiv \frac{1}{2}h^T_{\bar{m}\bar{n}\bar{p}} \pm \frac{1}{6}(\epsilon_{rsm}\rho_r h^T_{\bar{s}\bar{n}\bar{p}} + \epsilon_{rsn}\rho_r h^T_{\bar{s}\bar{m}\bar{p}} + \epsilon_{rsp}\rho_r h^T_{\bar{s}\bar{m}\bar{n}}). \tag{12}$$

Analysing the field equation (3), we find that $h^{T-}_{\bar{m}\bar{n}\bar{p}} = 0$ for $\mu > 0$. An iterative process yield

$$(\Box - \frac{\mu^2}{9})h^{T+}_{\bar{m}\bar{n}\bar{p}} = 0, \tag{13}$$

i.e. our action describes a single pure massive spin-3^+ excitation.

3. Spontaneous break-down of translational gauge invariance

We want to analyse the possibility of breaking down the (translational)local gauge invariance. We introduce a Fierz-Pauli spin-3 mass term $\sim m^2hh$ and consider the following action

$$\begin{aligned} I &= \frac{1}{2} < h_{m\bar{a}\bar{b}}G_{m\bar{a}\bar{b}} > + \frac{\mu}{2} < h_{m\bar{a}\bar{b}}\epsilon^{mnp}\partial_n h_{p\bar{a}\bar{b}} > \\ &- \frac{1}{6}m^2 < \epsilon^{pmn}\epsilon^{abc}\eta_{pc}h_{m\bar{b}\bar{d}}h_{n\bar{c}\bar{d}} > + \mu^2 < h_p v_p > \\ &+ \frac{\mu}{2}\alpha < v_p\epsilon^{pmn}\partial_m v_n > + \frac{\mu^2}{2}\beta < v_p^2 > \\ &+ \mu < \phi\partial_p v_p > + \frac{\mu^2}{2}\delta < \phi^2 > + \frac{1}{2}\gamma < \phi\Box\phi > . \end{aligned} \tag{14}$$

where we have taken for simplicity an algebraic(non-gauge invariant) coupling term $\sim \mu^2 h.v$ instead of the (gauge invariant) differential one we introduced in the initial action.

It is straightforward to prove, performing a covariant analysis that

$$\alpha = -18(\frac{\mu}{m})^4 = (\frac{\mu}{m})^2\beta, \quad \gamma = 0, \quad \delta = \frac{m^4}{24(\mu^4 + \mu^2m^2)} \tag{15}$$

induce the vanishing of all lower spin and our system only propagates two spin-3 excitations represented by the traceless, transverse, parity sensitive parts of $h^T_{m\bar{a}\bar{b}}$ defined in Eq. 11. Their evolution equations are found to be

$$\begin{aligned} \Box h^{T+}_{\bar{m}\bar{a}\bar{b}} &+ \frac{1}{3}\mu(\Box)^{\frac{1}{2}}h^{T+}_{\bar{m}\bar{a}\bar{b}} - \frac{1}{9}m^2h^{T+}_{\bar{m}\bar{a}\bar{b}} = 0, \\ \Box h^{T-}_{\bar{m}\bar{a}\bar{b}} &- \frac{1}{3}\mu(\Box)^{\frac{1}{2}}h^{T-}_{\bar{m}\bar{a}\bar{b}} - \frac{1}{9}m^2h^{T-}_{\bar{m}\bar{a}\bar{b}} = 0, \end{aligned} \tag{16}$$

giving rise to two different masses

$$m_{\pm} = \frac{\mu}{6}[(1 + 18\frac{m^2}{\mu^2})^{\frac{1}{2}} \mp 1]. \tag{17}$$

corresponding to the two spin-$3^{\pm}$ excitations that action (14) propagates.

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